THE RELATION OF STIMULUS TO SENSATION. A REPLY TO MR. MAX MEYER'S CRITICISM ON PROF. C. LLOYD MORGAN'S PAPER.

By PROF. F. R. BARRELL.

The July number of this *Journal* contains a criticism by Mr. (? Prof.) Meyer on a recent paper by my colleague, Prof. Lloyd Morgan. It is stated that "the mathematical discussion contains several errors;" as I was responsible for a certain amount of advice in the preparation and for a final confirmation of the expression of Prof. Morgan's views, I have volunteered to reply to the criticism.

Mr. Meyer ought to have recognized that the whole sting of his criticism depends on a question of Definition. A circle, ellipse, or hyperbola has each its definition consecrated by time: a logarithmic curve, sine curve or the like has not. For the purpose of the paper under discussion we defined a 'logarithmic curve' [and the definition is obvious from the context][1] as one in which the "ordinates are in Geometrical Progression when the abscissae are in Arithmetical Progression" or *vice versa*: that is, the curve represented either by $y=Ae^{Bx}$ or by $y=A \log Bx$.

This curve is a direct expression of the Weber-Fechner law, it stands in a definite relation to the axes of co-ordinates—*i. e.*, in our case to the lines of no-sensation and no-stimulus—it has the essential and to my mind objectionable property that it cannot pass through the origin and it cannot embrace the obvious fact that no-stimulus is accompanied by no-sensation.

Further, the language of Prof. Morgan's paper distinguishes between a 'part of a curve' and the 'complete curve:' this is in accord with my own habit; thus, I would not call a Gothic arch circular though it be composed of arcs of circles; nor would I call the curve of cosines the curve of sines, though it be merely the same curve in a different phase. This habit may not be as general as we imagined, and I would not seek to impose it on Mr. Meyer.

Prof. Morgan enunciates the law A "*Equal increments of*

[1] Prof. Morgan's phrase "*logarithmic, as it should be, if the Weber-Fechner formula holds good*;" may fairly be claimed as an explicit statement of the definition, for the curve $y=Ae^{Bx}+C$ is *not* 'as it should be,' if the Weber-Fechner law be true.

sensation are produced by increments of excitation in Geometrical Progression."

The Weber-Fechner Law B may be tersely enunciated "*Sensations increasing by equal increments are produced by excitations in Geometrical Progression.*"

Mr. Meyer overlooks the fact that law A is merely a particular case of law B, and, moreover, that it is the only case which cannot possibly hold good in the lower stages of sensation.

In my note to Prof. Morgan's paper[1] I showed how the mathematical expression of his law led to the Differential Equation

$$\frac{dy}{dx}\,\frac{d^3y}{dx^3} = \left(\frac{d^2y}{dx^2}\right)^2 \quad . \quad . \quad . \quad (I)$$

it is easy to show that the Weber-Fechner law leads to the equation

$$y.\,\frac{d^2y}{dx^2} = \left(\frac{dy}{dx}\right)^2 \quad . \quad . \quad . \quad (II)$$

Mr. Meyer must know the fundamental distinction between an equation of the third order and one of the second: viz.—the solution of one contains *three arbitrary constants*, that of the other contains *two*.

Thus

$$\text{(I) leads to } y = Ae^{Bx} + C = A10^{bx} + C \quad . \quad . \quad . \quad (III)$$

$$\text{(II) leads to } y = Ae^{Bx} = A10^{bx} \quad . \quad . \quad . \quad (IV)$$

the similarity between these two results, and the accident that by putting C=o, or by adjusting the axes, III reduces to IV, ought not to blind us to the above essential difference; nor ought we to be influenced by the fact that any arc of III can be superposed in Euclidean fashion upon the corresponding arc of IV.

The advantage of possessing three arbitrary constants A, B, C as in (III) is seen as follows:—Since any arbitrary scales may be chosen for representing stimulus and sensation, it is convenient in each experiment to represent the extreme sensation and extreme stimulus each by 100: Thus III must be satisfied by y=100, x=100:

$$\therefore\ 100 = A10^{100b} + C \quad . \quad . \quad . \quad (V)$$

further, it is axiomatic that 'no-stimulus' is accompanied by 'no-sensation' [this may be a psychological blunder; if so, as a

[1] Psychological Review, VII, No. 3, pp. 217-233, 1900.

mathematician I must be pardoned], hence III must be satisfied by y=o, x=o

$$\therefore \quad 0=A10^0+C \quad \therefore \quad c=-A$$

which by substitution in (V) gives

$$100=A10^{100b}-A$$

$$i.\ e.\ 100=A\ (10^{100b}-1) \quad . \quad . \quad . \quad (VI)$$

this single relation between A and b leaves at our disposal one other relation betweeu them, and this we utilize to give character to the sensation under discussion by making the curve pass through any single point determined in the experiment: in the case of White on 'Morgan's Black' this was the point y=27, x=50 giving

$$27=A\ (10^{50b}-1) \quad . \quad . \quad . \quad (VII)$$

in the case of Red on 'Morgan's Black' it was y=35, x=50, giving

$$35=A\ (10^{50b}-1)$$

Equations VI and VII determine the values A=15.85, b=.008639.

If we attempt to treat formula IV in the same way, we begin as before:—put y=100, x=100

$$\therefore\ 100=Ae^{100b} \quad . \quad . \quad . \quad (V^1)$$

now put y=o, x=o

$$\therefore\ 0=A10^{0.b} \quad . \quad . \quad . \quad (VI^1)$$

Unfortunately this cannot be satisfied by finite values of A and b, and we are confronted by the fundamental objection to the Weber-Fechner law—it cannot be applied to the low stages of sensation. But supposing even that (VI^1) could be satisfied by finite values of A and b, then V^1 and VI^1 between them would use up both the available constants, and we should have no constant left with which to give character to the particular sensation under investigation.

This point is brought out clearly by Prof. Morgan in his Fig. 4 in a comparison (which Mr. Meyer erroneously condemns) between the curve derived from his own experiment and the 'logarithmic-curve-as-it-should-be-if-the-Weber-Fechner-formula-holds-good,' which fits it most closely.

This family of curves being represented by $y=A10^{bx}$, any member of the family IS completely determined by two given points. How were these two points to be selected? As explained above, the desirable point y=o, x=o had of necessity to be abandoned: then in order to secure some degree of proximity between the curves the other extreme point y=100, x=100 was abandoned: finally, as clearly stated by Prof. Morgan and *quoted by Mr.*

Meyer the stages 6 and 14 were selected for coincidence: from the tabulated results we have

at stage 6 $y=13$, $x=30$ $\therefore$ $13=A10^{80b}$
at stage 14 $y=48$, $x=70$ $\therefore$ $48=A10^{70b}$.

These give $A=4.881$, $b=.01418$, hence the equation to the broken curve is $y=4.881\times 10^{.01418x}$ and, on the assumption that Prof. Morgan's black gives a reasonably approximate zero of stimulus as a basis for calculation, the contrast between this broken curve and the experimental curve illustrates well the difference between the new law and the old.

The latter part of Mr. Meyer's paper is interesting, as it brings out the simple mathematical fact that any member of the family of curves $y=A(10^{bx}-1)$ may be converted into any other one by a suitable increase or decrease of both the horizontal and vertical scales of representation: this property is so familiar in the case of ellipses and certain other curves that it did not seem necessary to call attention to it. Moreover, I cannot see that Mr. Meyer's method of comparing the results 'White on Black,' 'Red on Black,' 'Blue on Black' by adjusting the scales of the second and third so as to make them coincide with positions of the first, is intrinsically superior to Prof. Morgan's method of comparison by means of three distinct curves bridging over the same interval.

Mr. Meyer's method may, however, be the better if his suggestion be correct that the thing dealt with in the experiment was intensity of illumination rather than toning of color; this idea had previously suggested itself to Prof. Morgan, who said in his paper "so far as the colors are concerned the results appear to be due rather to the relative intensities of stimulation or excitation of the retina than to the effects of color as such."

The only fault which I can detect in Prof. Morgan's exposition of his results is that he neglected to point out the patent fact that by assuming (without any experimental basis for the assumption) that his black contained precisely 15.8% of white and 40.8% of his red (or of their illuminations) he would bring his results in line with the Weber-Fechner formula: this omission left it open to Mr. Meyer to assume that the fact had not been noted.

In conclusion I must add that even if the law "*Equal increments of sensation are produced by increments of excitation in Geometrical Progression*" be merely considered as a modified statement of Weber's law, yet it is a very material improvement on the old mode of expression, for it deals with the only things which are actually estimated in these and similar experiments, where the true zero of stimulus is difficult to ascertain, namely, *increments* of stimulus and *increments* of sensation.

Further, although the formula $y=A\ (10^{bx}-1)$ may not stand the ultimate test of being applied to low stages of sensation in cases where the zero of stimulus can be absolutely ascertained (as, *e. g.*, in estimating saltness of dilute solutions) yet it has the *prima facie* advantage that it permits us to work downwards without break of continuity to the point "no-stimulus, no-sensation," and that, as Prof. Morgan pointed out in his paper, it gives near the threshold of sensation an arithmetical progression of stimulus, which accords with results obtained by the pupils of Hering.

I am, of course, aware of the accepted opinion that starting with no-stimulus, and gradually introducing stimulus, some finite stimulus must be reached before any sensation is recognized; this seems to indicate some discontinuity at the inception of sensation, and might necessitate a corresponding discontinuity in the formula representing it. Yet, although some such explosive origin may be possessed by sensation, the hiatus between the zero and the starting point of sensation must be so slight, that it is desirable if possible in this, as in physical phenomena, to bridge over the hiatus by a formula which will extend down to the origin.